1908MNRAS..68..490S

On Dr. Roberts' method of determining the absolute dimensions of an Algol Variable Star. By the Rev. J. Stein, S.J., Sc.D.

(*Communicated by Professor H. H. Turner.*)

In an interesting article of the January number, 1906, of the *M.N.* (vol. lxvi. p. 123), Dr. Alex. Roberts starts from the hypothesis that the light-changes of the Algol variables U Pegasi and RR Centauri are fairly well explained by two eclipsing stars moving in a circular orbit, and tries to reduce the residuals (O—C) by applying some corrections to the theoretical light-curve. He supposes that the orbit is passing through the line of sight, and puts—

P = Period of variable in minutes.
ϵ (very small) = Eccentricity of Orbit.
λ = Longitude of periastron, reckoned from line of sight.
θ = True angular distance of eclipsing star from line of sight at time T.
Δm = Change in magnitude per minute at time T.
l = Time (in minutes) that light takes to cross the semi-orbit.
Δt = Correction to the assumed time of passing the principal minimum phase.
$p = \epsilon \sin \lambda,\ q = \epsilon \cos \lambda$.

Then, neglecting the second and higher powers of ϵ, he proceeds from the typical equation of condition—

$$\Delta m \cos \theta \frac{2}{n} p + \Delta m \sin \theta \frac{2}{n} q + \Delta m \, \Delta t + \Delta m |\cos \theta| l + \ldots = (\mathrm{O—C}),$$

where p, q, Δt, l are unknown quantities, $n = \frac{2\pi}{P}$ and $|\cos \theta|$ denotes the absolute value of $\cos \theta$.

According to the author, the three first terms are corrections depending on eccentricity and variation of epoch; the fourth term is the correction due to the fact that light takes an appreciable interval of time to traverse the orbit. (The other corrections will here be left out of consideration.) Hence, resolving the equations of condition after the method of least squares, Dr. Roberts finds l, and consequently the absolute dimensions of the orbit.

It seems, however, that the point of departure of this method is open to criticism. Indeed, the first member of the fundamental equation is not the adequate expression of the corrections due to eccentricity, variation of epoch, and aberration of light.

To show this, let us take the most simple case—that the two components are both spherical, and that the nearly circular relative orbit is passing through the line of sight. In this case the apparent brightness of the binary system is only depending on the apparent distance ρ of the centres of the two spheres, as seen from our Sun.

1908MNRAS..68..490S

Hence we may put—

Change in magnitude in theoretical light-curve deduced from circular orbit,

$$\Delta m = \frac{dm}{d\rho}\frac{d\rho}{dt} \quad . \quad . \quad . \quad . \quad (1)$$

For the purpose of determining the effect of eccentricity on ρ, we may neglect the aberration of light. Thus, in the circular orbit we have—

$$\rho^2 = r^2 \sin^2 \theta \quad . \quad . \quad . \quad . \quad (2)$$

r denoting the constant radius-vector, and θ referring to the star S_1 eclipsing at principal minimum.

Consequently, from (1) and (2),

$$\frac{dm}{d\rho} = \Delta m \tan \theta \frac{1}{n\rho} \quad . \quad . \quad . \quad . \quad (3)$$

Putting $\Delta\rho$, Δr, $\Delta\theta$ = the variations of ρ, r, θ, at the time T by passing from the circular orbit to the improved, eccentrical one, we find—

$$\frac{\Delta\rho}{\rho} = \frac{\Delta r}{r} + \cot \theta \, \Delta \theta \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{\Delta r}{r} = -\epsilon \cos (\theta - \lambda) = -q \cos \theta - p \sin \theta \quad . \quad . \quad . \quad . \quad (5)$$

$$\Delta\theta = 2\epsilon \sin (\theta - \lambda) + \text{const.}$$

For $\theta = 0$ we shall have $(\Delta\theta)_0 = n \, \Delta t$; consequently

$$\begin{aligned}\Delta\theta = 2\epsilon \sin (\theta - \lambda) + 2\epsilon \sin \lambda + n \, \Delta t = 2q \sin \theta& \\ + 2p (1 - \cos \theta) + n \, \Delta t& \quad (6)\end{aligned}$$

Substituting (5) and (6) in (4),

$$\frac{\Delta\rho}{\rho} = q \cos \theta - p \frac{(1 - \cos \theta)^2}{\sin \theta} + n \cot \theta \, \Delta t \quad . \quad . \quad (7)$$

Finally, according to (3) and (7), the adequate variation of magnitude at the time T, due to eccentricity and variation of epoch, will be

$$\frac{dm}{d\rho} \Delta\rho = \Delta m \sin \theta \frac{1}{n} q - \Delta m \frac{(1 - \cos \theta)^2}{\cos \theta} \frac{1}{n} p + \Delta m \, \Delta t \quad (8)$$

Vice versa, for determining the effect of aberration, we may neglect the eccentricity of the orbit, and put—

a_1, a_2 = the semi-axes of the circular orbits of the two components S_1 and S_2 around the common centre of gravity.

ρ_1, ρ_2 = the apparent projections of the radii vectores on the tangential plane of the celestial sphere at the time T, this being reckoned from the moment of principal minimum.

l_1, l_2, σ = Time (in minutes) that light takes to cross the semi-

1908MNRAS..68..490S

axes of the orbits and the distance from the centre of gravity to the Sun.

Then the time taken by the light—

to pass from S_1 to the Sun will be $= \sigma - l_1 \cos n(T - \sigma)$

" " S_2 " " $= \sigma + l_2 \cos n(T - \sigma)$

Consequently, putting $n(T - \sigma) = \theta$:

$$\rho_1 = a_1 \,|\sin(\theta + nl_1 \cos\theta)|$$
$$\rho_2 = a_2 \,|\sin(\theta - nl_2 \cos\theta)|$$

where cos θ is to be taken with its proper sign.

The quantities l_1 and l_2 being generally very slight, and since $a_1 l_2 = a_2 l_1$, we may put—

$$\rho = \rho_1 + \rho_2 = a\,|\sin\{\theta + n(l_1 - l_2)\cos\theta\}| \quad . \quad (9)$$

and consequently the correction for light-equation will be

$$\Delta m(l_1 - l_2)\cos\theta \quad . \quad . \quad . \quad (10)$$

Thus it would seem theoretically impossible to determine the absolute dimensions of the orbit in this manner. If this statement is correct, it follows that the numerical results of Dr. Roberts' paper cannot have the interpretation which is given to them.

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Note to Father Stein's paper. By Professor H. H. Turner, D.Sc., F.R.S.

The following elementary geometrical considerations indicate the reasons why the light-equation fails to give us expected information about the orbit. They were mentioned at the meeting of the Society in communicating Father Stein's paper, and are added here as a note at the request of one of the Secretaries.

Consider the three cases—(*a*) when the mass of the bright body is so small as to be negligible; (*b*) when the mass of the dark body is negligible; (*c*) when the two bodies are of equal mass. And in each case let us suppose the orbit circular and in a plane passing through the line of sight, and that light takes a time 2T to cross the orbit.

(*a*) In the first case (fig. 1) the dark body is stationary at the centre O, and the bright body revolves round it, and is eclipsed at A, the Earth being in the direction of the arrow. The light which suffers eclipse reaches O at a time T *after* the actual eclipse.

(*b*) In the second case (fig. 2) the bright body is stationary at the centre O, and the dark body revolves round it and eclipses it when at B. But the light which suffers eclipse must leave O at a time T *before* the actual eclipse.